higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1010**(E)**(A2)T

NATIONAL CERTIFICATE

MATHEMATICS N3

(16030143)

2 April 2019 (X-Paper)
09:00–12:00

This question paper consists of 7 pages and a formula sheet of 2 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order but keep subsections together.
5. Show ALL calculations and intermediate steps.
6. Do ALL graph work in the ANSWER BOOK. NO graph paper needed.
7. Accurately approximate ALL final answers rounded off to THREE decimal places.
8. Diagrams are NOT drawn to scale.
9. Write neatly and legibly.

QUESTION 1

1.1 Determine the value of a if $f(x)=x^3+ax^2-x+5$ is divided by $x-2$ and gives a remainder of 23. (3)

1.2 Simplify each of the following:

1.2.1 $$\frac{y(x+y^2+y)+x}{y+1}\div\frac{4y^2-3yx-7x^2}{y^2-x^2}\times\frac{4y-7x}{x+y^2}$$ (7)

1.2.2 $$\frac{6}{x-2}+\frac{3}{2+x}-\frac{9x-5}{x^2+x-6}$$ (7)

[17]

QUESTION 2

2.1 Solve for x:

$$\log_2(x+1)=\log_{(x+1)}2$$ (6)

2.2 Simplify the following expressions:

2.2.1 $$\frac{a^{\frac{1}{2}}b^{\frac{1}{2}}-b}{a-b}\div\left(1+\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}}\right)^{-1}$$ (6)

2.2.2 $$\frac{\log_a 16-\log_b 4}{\log_a 4-\log_b 2}$$ (4)

[16]

QUESTION 3

3.1 When 1 is added to the numerator and the denominator of a fraction, the fraction becomes $\frac{1}{2}$. When 2 is subtracted from the numerator and the denominator of the same fraction, the fraction becomes $\frac{1}{5}$.

Determine the fraction. (6)

3.2 Make R the subject of the following formula:

$$\frac{RMN}{RE + MA} = 1$$ (3)

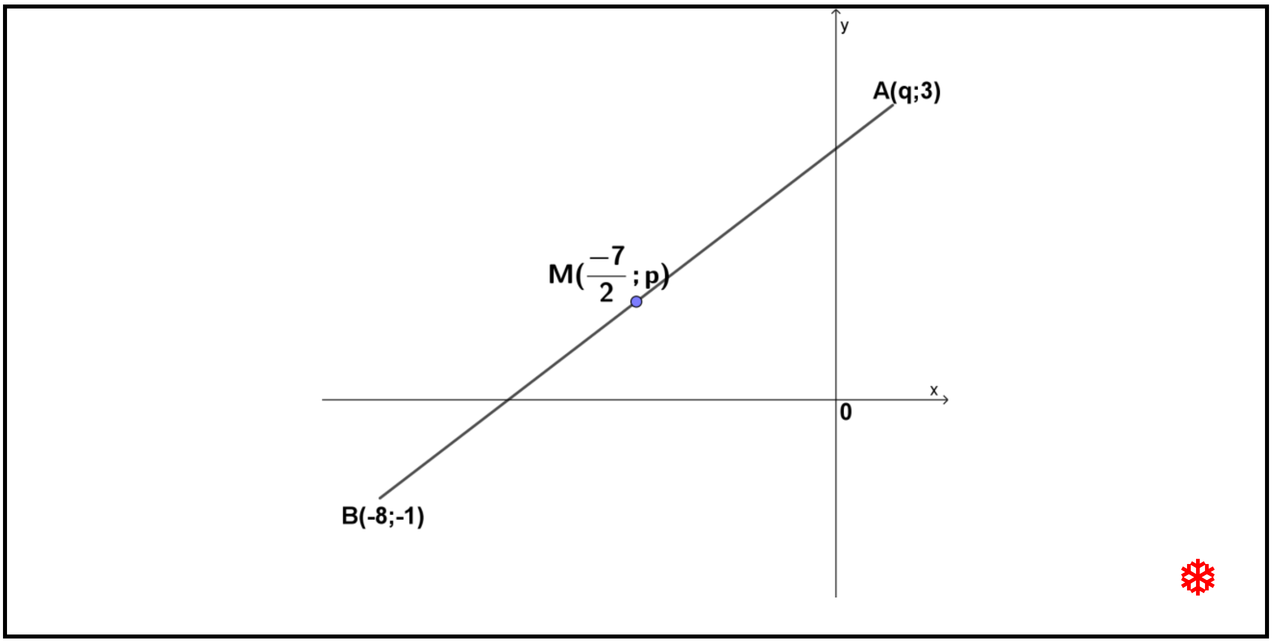
3.3 Solve for x by completing the square:

$$4x^2 = 4x + 1$$ (5)

[14]

QUESTION 4

4.1 Calculate the value of p and q if M is the midpoint of line segment AB in FIGURE A below.



(3)

FIGURE A

4.2 The vertices of a triangle are given as $O(0;0)$, $B(\sqrt{2};\sqrt{2})$ and $C(x;-\sqrt{2})$.

4.2.1 Prove that the value of $x = \sqrt{2}$ if $B\hat{O}C = 90°$ (3)

4.2.2 Determine the perimeter of the triangle. (4)

4.3 Proof that the straight line $3y - x - 10 = 0$ is a tangent to the circle $x^2 + y^2 = 10$. Find also the point of contact.

Hint: the general equation $x_1 x + y_1 y = r^2$ and $x = \dfrac{-mc \pm \sqrt{r^2 + m^2 r^2 - c^2}}{m^2 + 1}$ may be used. (5)

4.4 Determine the equation of the line through the point $P(2;-3)$ and parallel to the y-axis. (1)

4.5 Consider FIGURE B. $A(-1;2)$, B and $C(1;-1)$ are three points on the cartesian plane with $B\hat{A}C = 78{,}69^{\circ}$ and B a point in the third quadrant. Determine the equation of AB.

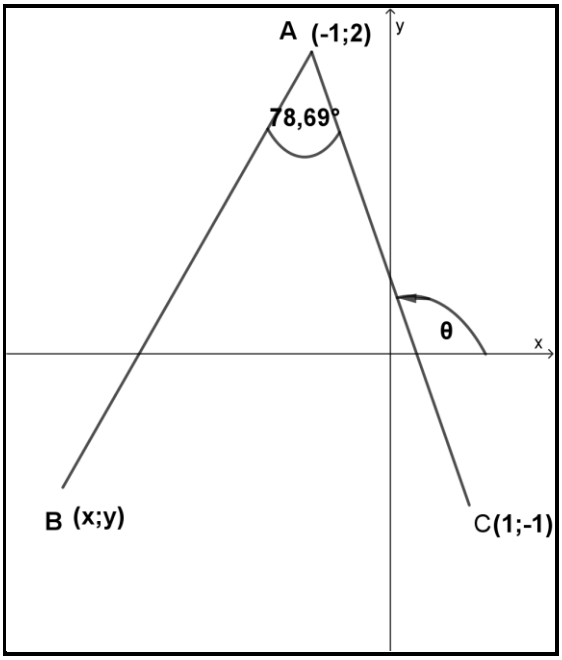


FIGURE B

(6)

[22]

QUESTION 5

5.1 Sketch the graph of $25x^2 + 4y^2 = 100$ in the ANSWER BOOK. Indicate clearly any intercepts with the axes. (3)

5.2 Determine $\dfrac{dy}{dx}$ of the following function by using the rules of differentiation. Leave the final answer with a positive exponent and in surd form.

$$y = \frac{\sqrt{x}}{4} - \frac{1}{4x^2}$$ (5)

5.3 FIGURE C shows the graph $f(x) = x^3 + bx^2 + cx$. A and $B(3;0)$ are turning points.

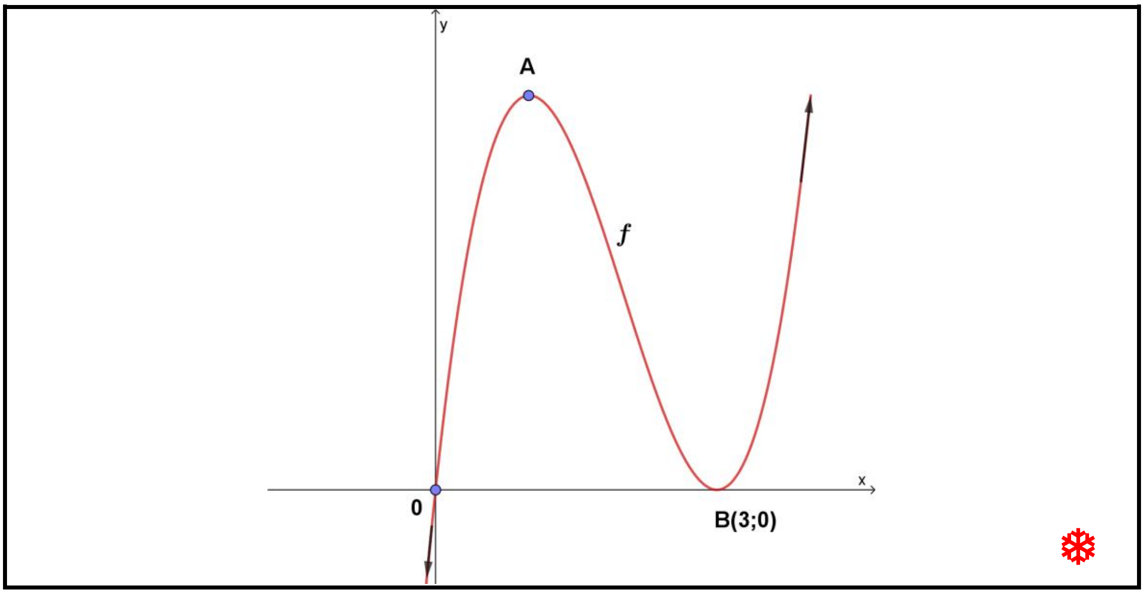


FIGURE C

5.3.1 Proof that the values of $b = -6$ and $c = 9$ (2)

5.3.2 Determine the coordinates of the maximum turning point A. (4)

[14]

QUESTION 6

6.1 Use trigonometric identities to prove that:

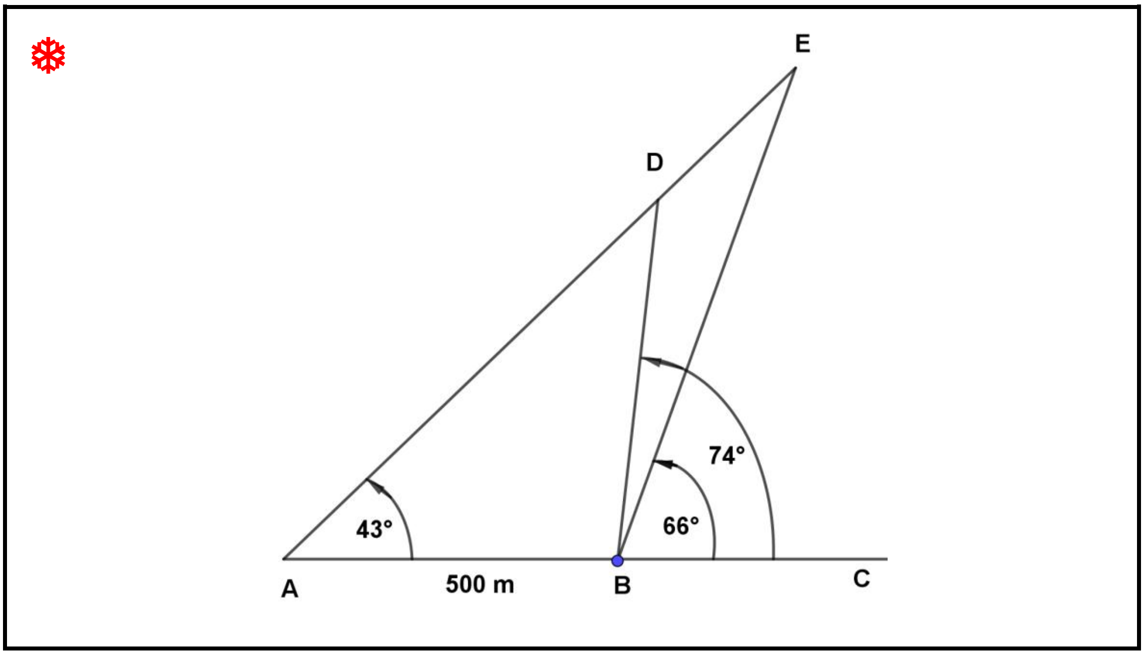
$$\frac{1-(\sin\theta - \cos\theta)^2}{\sin\theta} = 2\cos\theta$$ (4)

6.2 Calculate the value(s) of θ which will satisfy the equation if $0^\circ \leq \theta \leq 270^\circ$:

$$\sqrt{3}\tan\left(\theta + 10^\circ\right) = 3$$ (4)

6.3 Consider FIGURE D. A man decided to take a walk on a straight road that passes through points A and B towards point C. Before he takes the walk, he notices two cars, D and E, parked on his left-hand side. From where he stands he sees that the cars are on a straight line with point A where he stands. The angle between the straight line ADE and the road ABC is 43°. He walks to point B which is 500 m away from A where he was standing. From B he observes points D and E where he finds that the angles between BD and BC and between BE and BC are 74° and 66° respectively.

Determine the length of BD and DE.

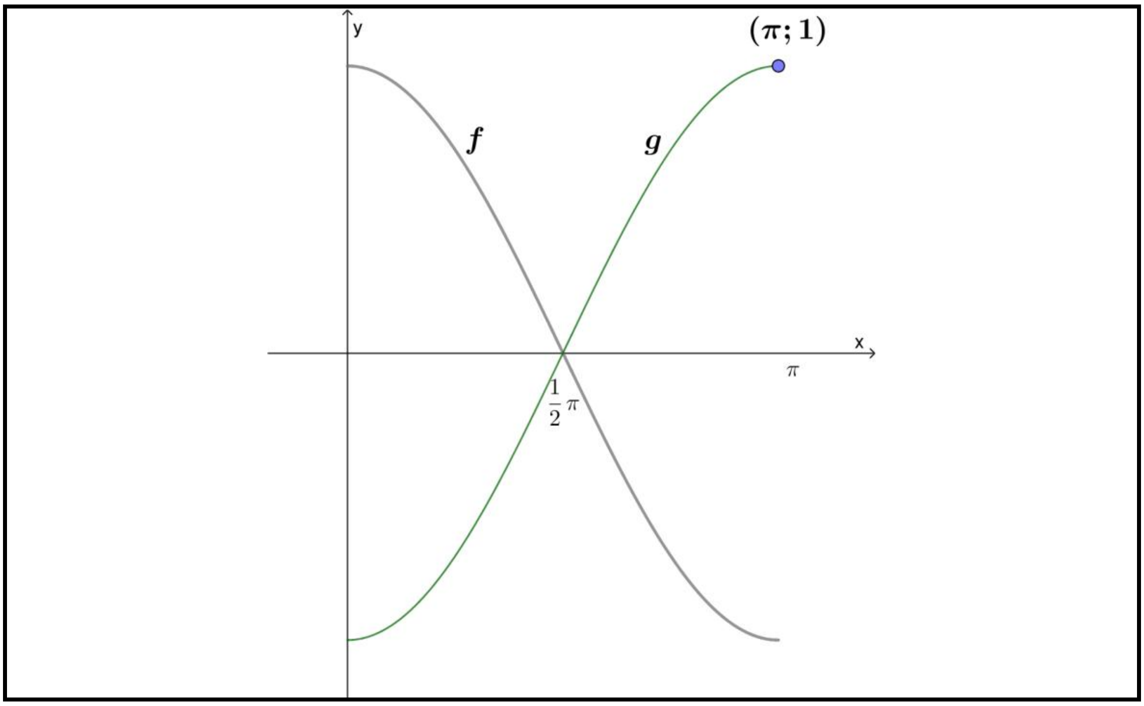


(7)

FIGURE D

6.4 FIGURE E below represents graphs of $f(x) = \cos ax$ and $g(x) = b\cos x$ where $0 \leq x \leq \pi$.

Determine the values of a and b.



(2)

FIGURE E **[17]**

TOTAL: 100

FORMULA SHEET

Any applicable formula may also be used.

1. Factors

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2. Logarithms

$\log ab = \log a + \log b$

$\log \dfrac{a}{b} = \log a - \log b$

$\log_b a = \dfrac{\log_c a}{\log_c b}$

$\log a^m = m \log a$

$\log_b a = \dfrac{1}{\log_a b}$

$\log_a a = 1 \therefore 1n\ e = 1$

$a^{\log_a t} = t \therefore e^{1n\, m} = m$

3. Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Parabola

$y = ax^2 + bx + c$

$y = \dfrac{4ac - b^2}{4a}$

$x = \dfrac{-b}{2a}$

5. Circle

$x^2 + y^2 = r^2$

$D = \dfrac{x^2}{4h} + h$

$x = \sqrt{4Dh - 4h^2}$

6. Straight line

$y - y_1 = m(x - x_1)$

Perpendicular: $m_1 \cdot m_2 = -1$

Parallel lines: $m_1 = m_2$

Distance: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint: $P = \left(\dfrac{x_1 + x_2}{2} ; \dfrac{y_1 + y_2}{2} \right)$

Angle of inclination: $\theta = \tan^{-1} m$

7. Differentiation

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$

Max/Min

For turning points: $f'(x) = 0$

8. Trigonometry

$$\sin\theta = \frac{y}{r} = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$